2017 May Maths eAssessment

Question 1 (10 marks)

Question 1a (2 marks)

Find 40 as a product of prime factors.

Question 1b (2 marks)

Consider two sets: A and B. Set A contains the factors of 40 and set B contains the multiples of 4 less than 21.

Determine the missing elements for set A and set B.

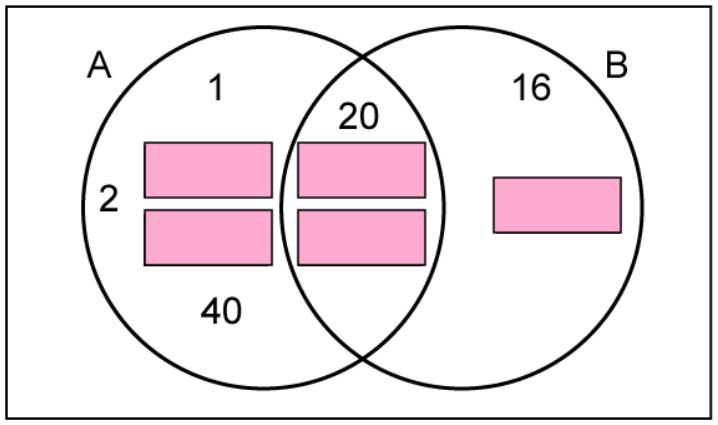
A = {1, 2, ☐, ☐, ☐, ☐, 20, 40}

B = {4, ☐, ☐, 16, 20}

Question 1c (3 marks)

Determine the missing values and complete the Venn diagram with the elements of sets A and B.

To insert your answers in the Venn diagram below, click inside the relevant box, then write the value in the "Add Label" box.



Question 1d (1 mark)

Write down the elements of A ∩ B.

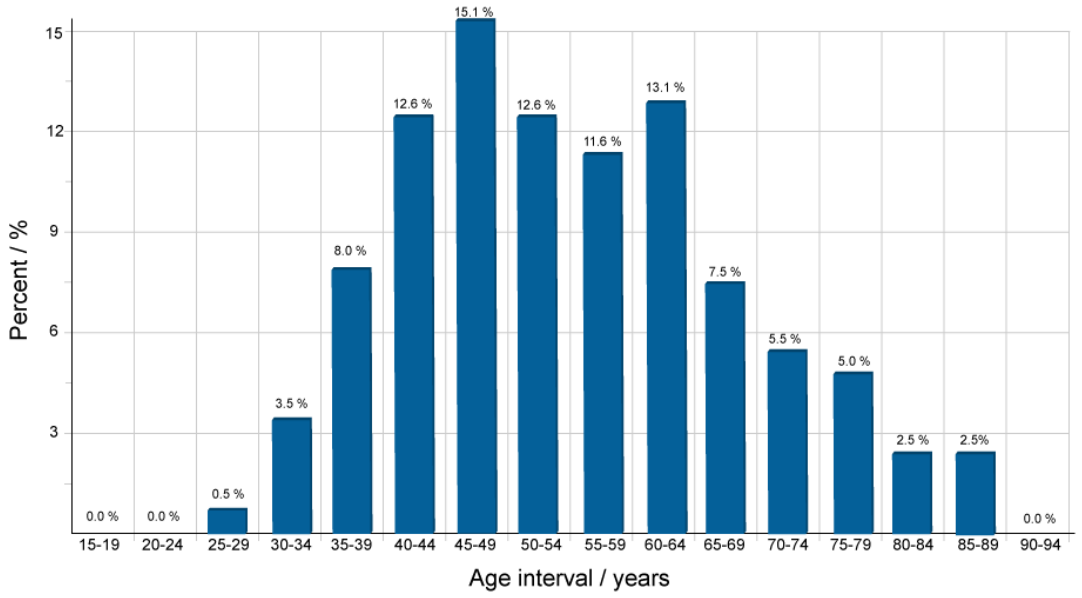
Question 1e (2 marks)

Describe the properties of the elements of A ∩ B in the context of the problem.

Question 2 (9 marks)

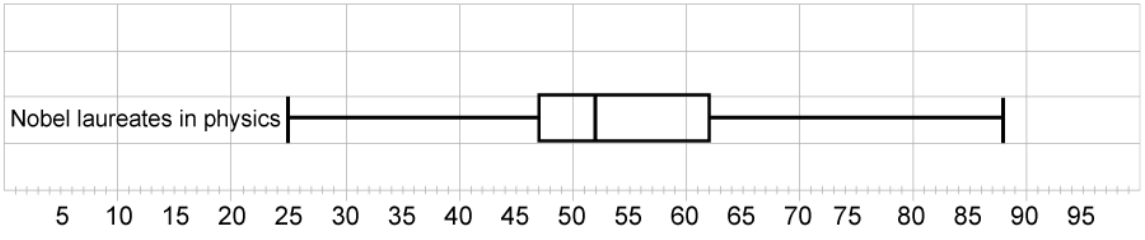
The Nobel Prize is a set of awards presented annually in recognition of academic, cultural and scientific advances. Every year, the Nobel Prize is awarded in recognition of discoveries made. Winners of a Nobel Prize are called Nobel laureates.

The bar chart below shows the distribution of the ages of the physics laureates the year they received their Nobel Prize. The data provided is taken from 2016.



Here is a box-and-whisker plot of Nobel laureates in physics. Hover over the box-and-whisker plot to reveal the values.

This media is interactive



Question 2a (1 mark)

Write down the modal interval for the age of the laureates of the physics award.

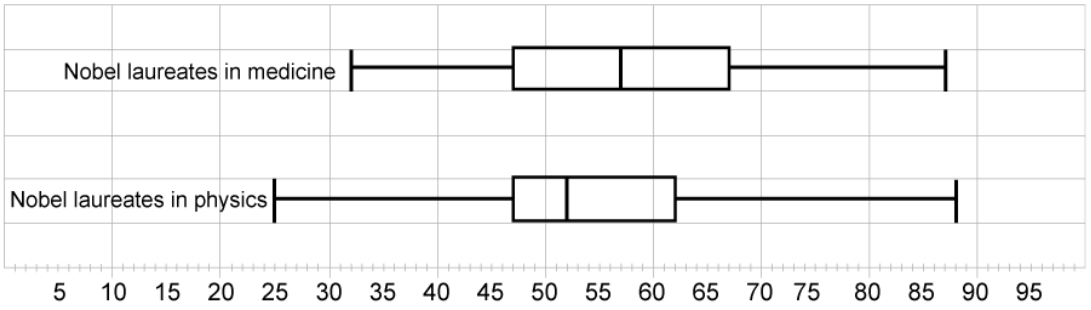
Question 2b (1 mark)

Write down the median age of the laureates of the physics award.

Question 2c (3 marks)

Here are the box-and-whisker plots of Nobel laureates in medicine and physics. Hover over each box-and-whisker plot to reveal the values.

This media is interactive

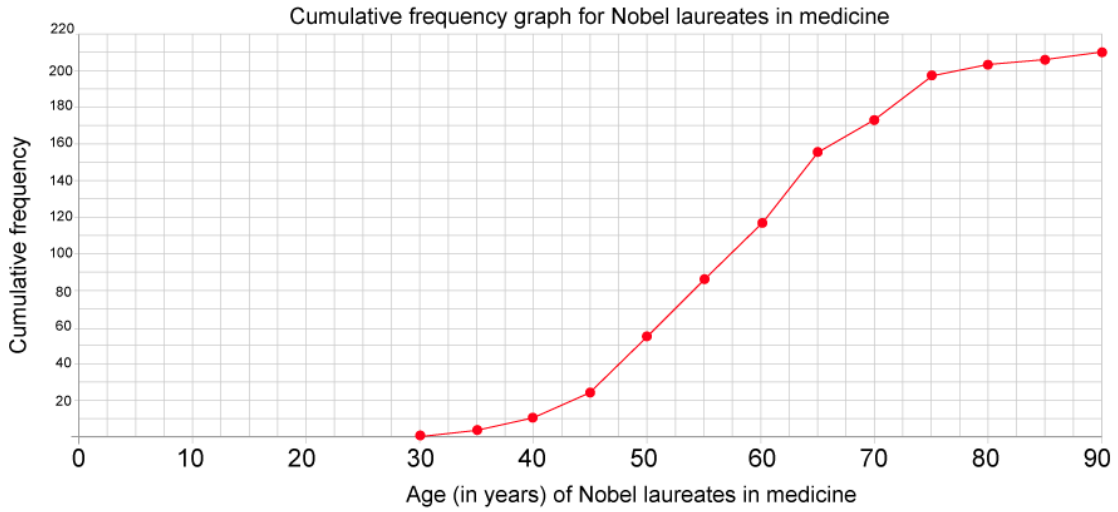


Compare the interquartile ranges of ages of the laureates of the medicine and physics awards.

Cumulative frequency graph | Cumulative frequency table

The cumulative frequency graph below shows the age (in years) of 210 Nobel laureates in medicine. Hover over the points to reveal the coordinates.

This media is interactive



Cumulative frequency graph | Cumulative frequency table

Age (in years) of Nobel laureates in medicine	Cumulative frequency
Under 30	0
Under 35	3
Under 40	12
Under 45	22
Under 50	55
Under 55	88
Under 60	118
Under 65	156
Under 70	172
Under 75	189
Under 80	201
Under 85	206
Under 90	210

Question 2d (2 marks)

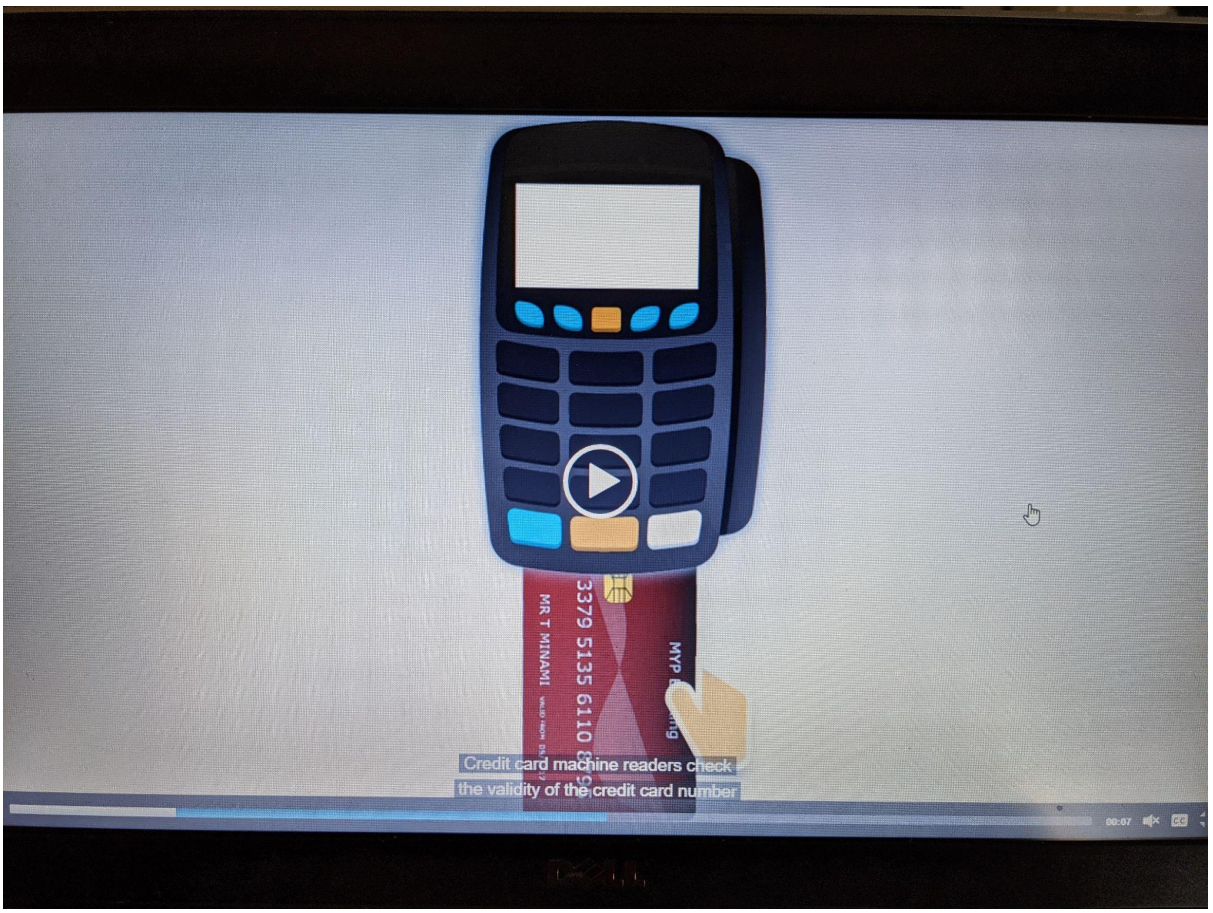
Using the cumulative frequency graph or table, **find** an estimate for the probability that the next Nobel laureate in medicine will be below 50 years old.

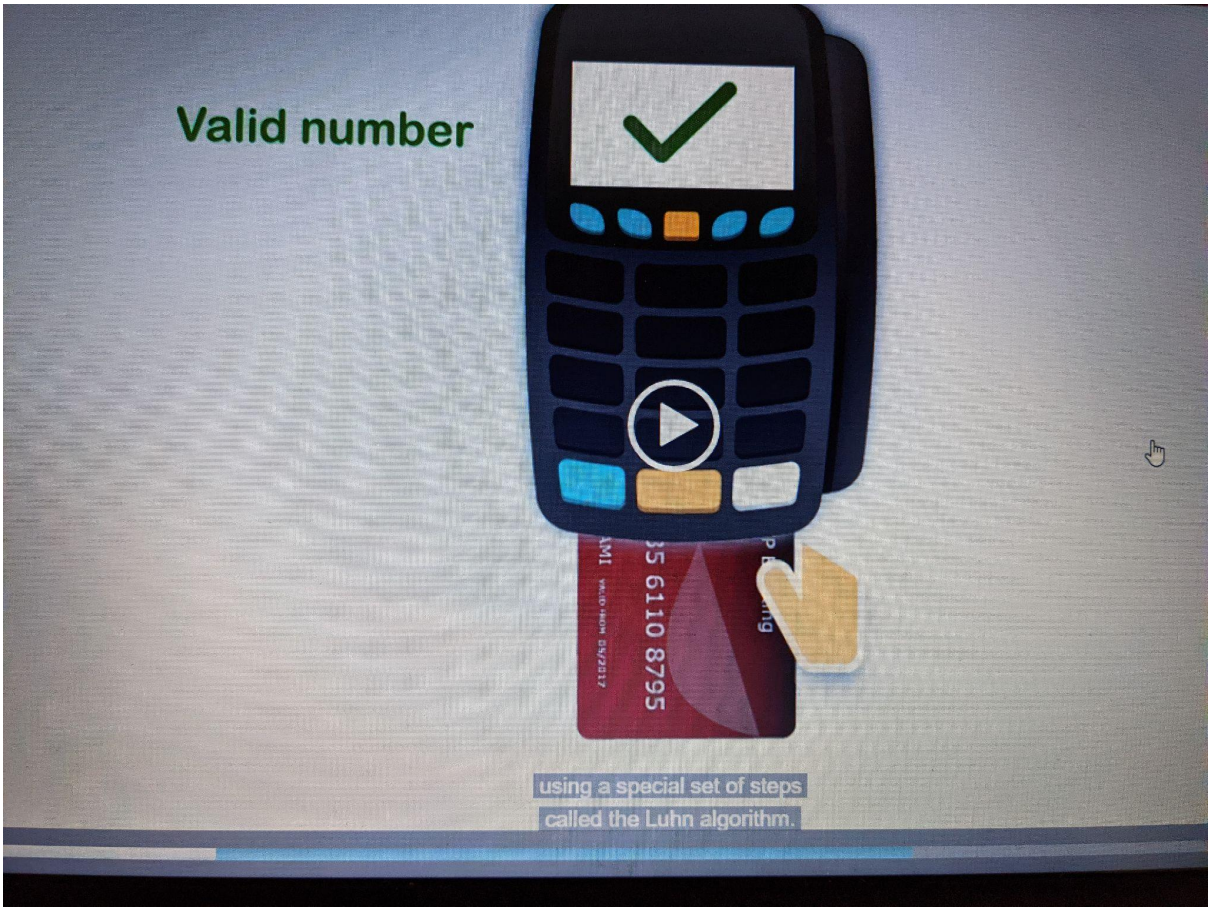
Question 2e (2 marks)

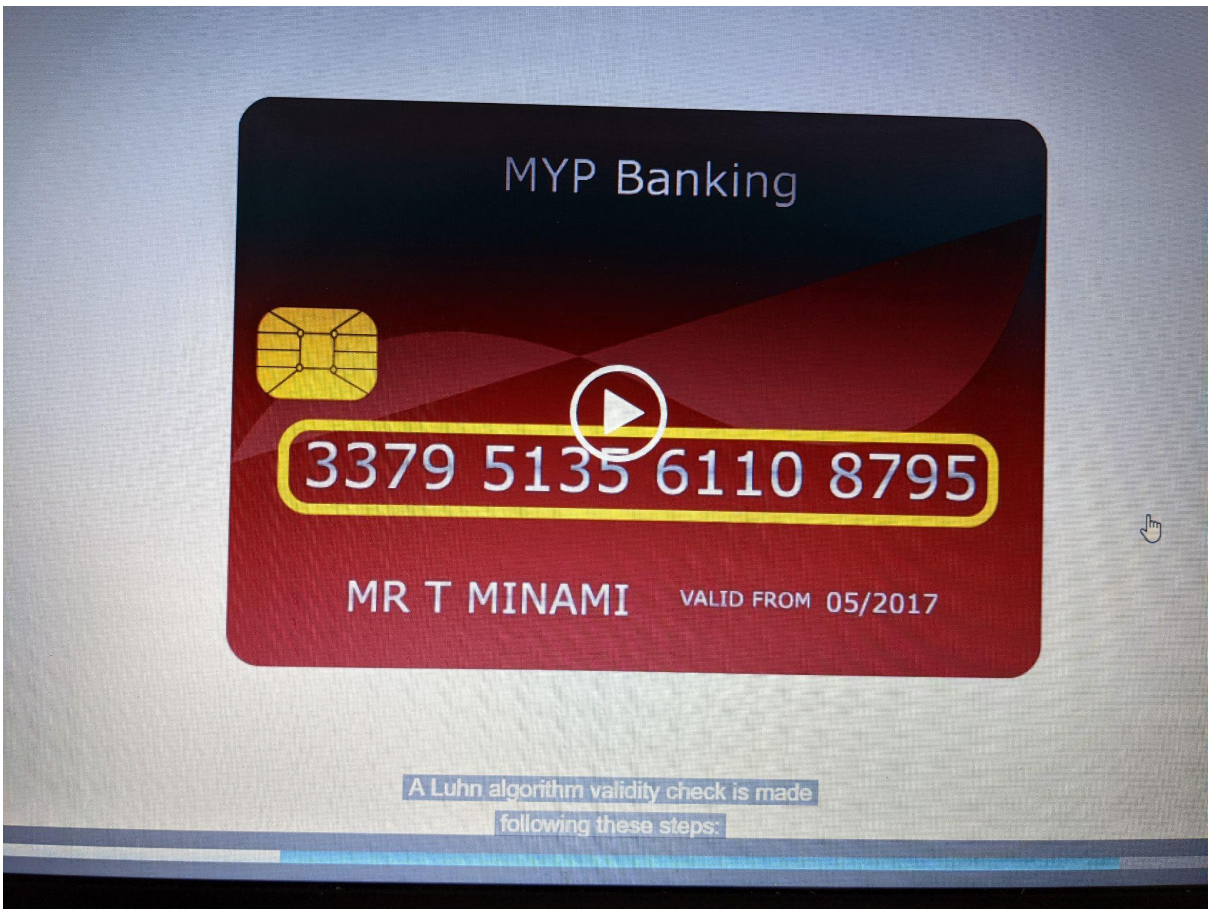
Given that 20 % of the Nobel Prize laureates in medicine are over x years old, **find** an estimate for the age x.

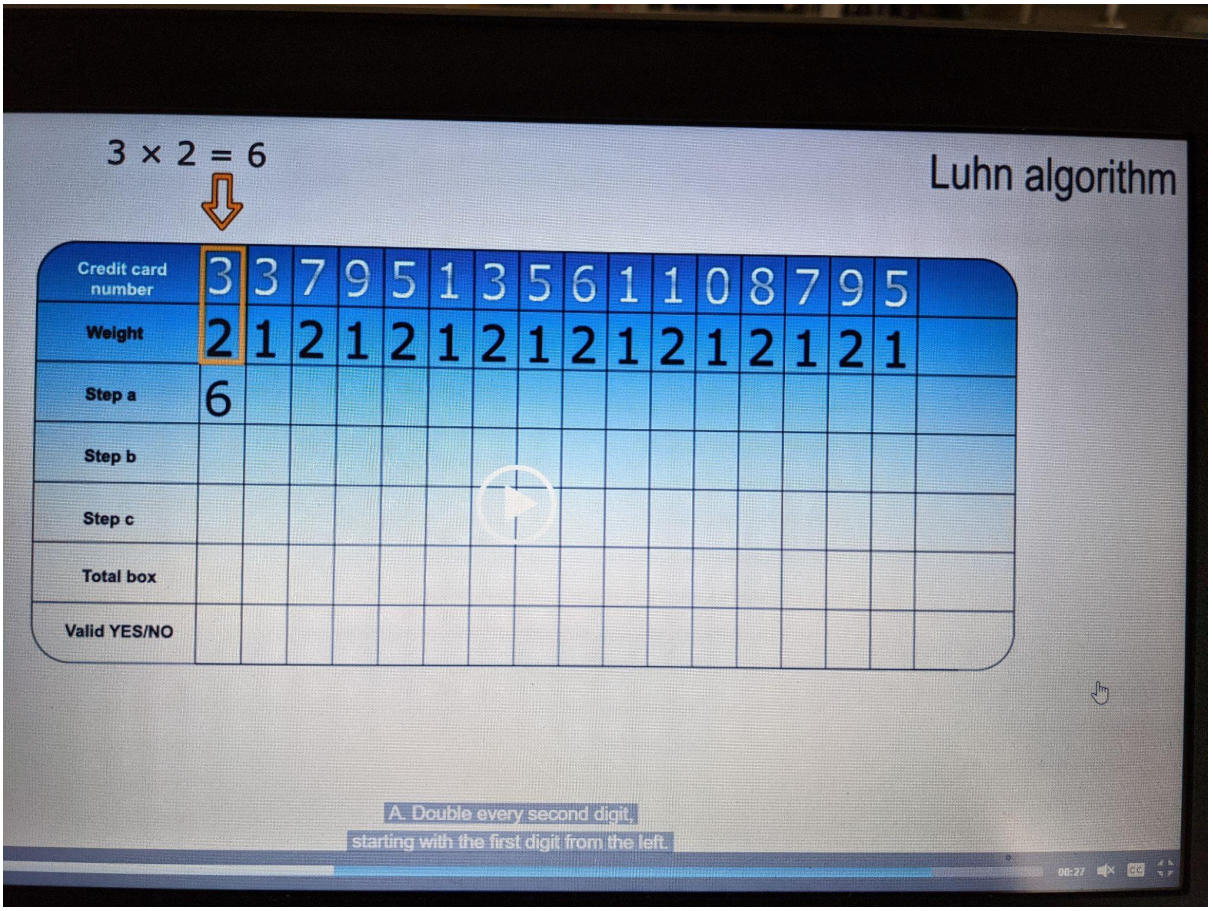
Question 3 (9 marks)

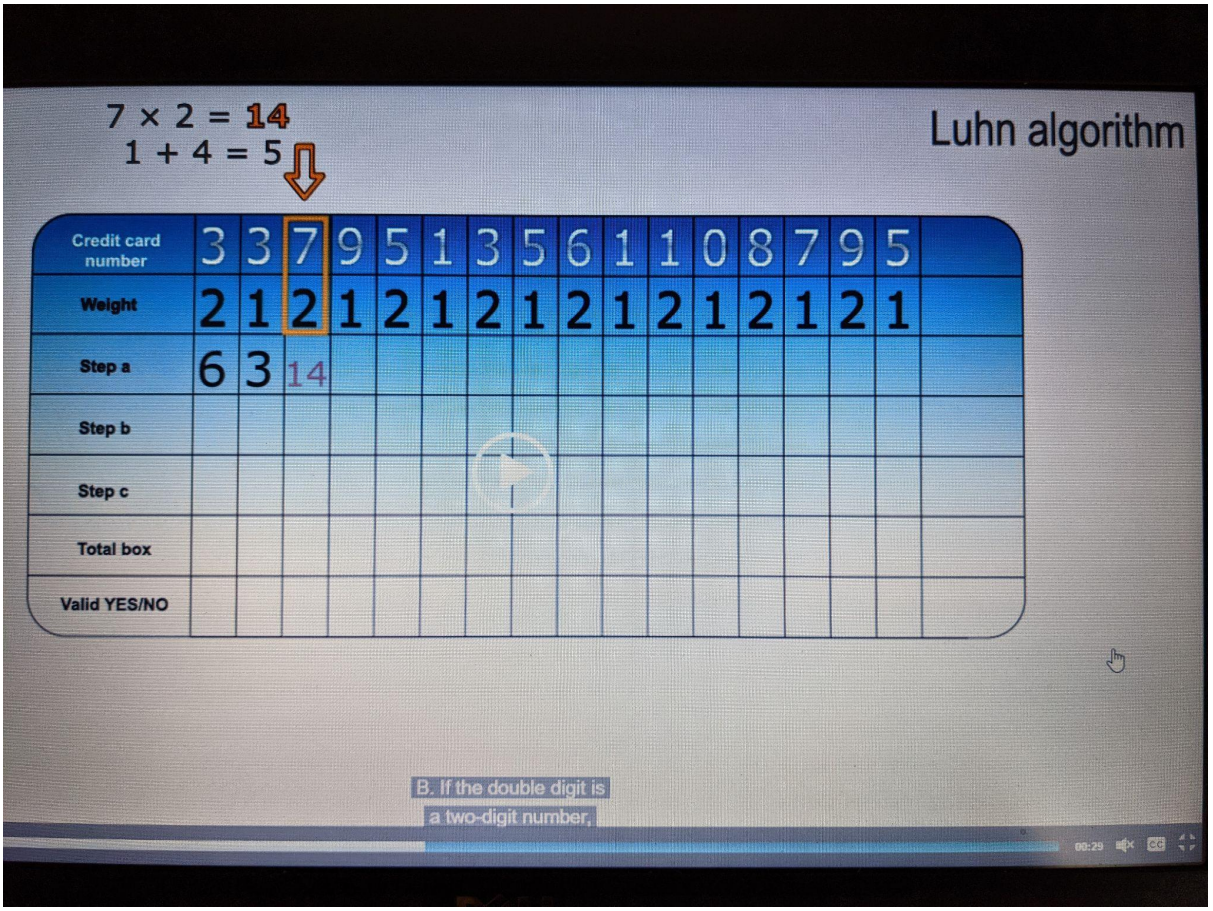
The following video describes the Luhn algorithm credit card validity check.

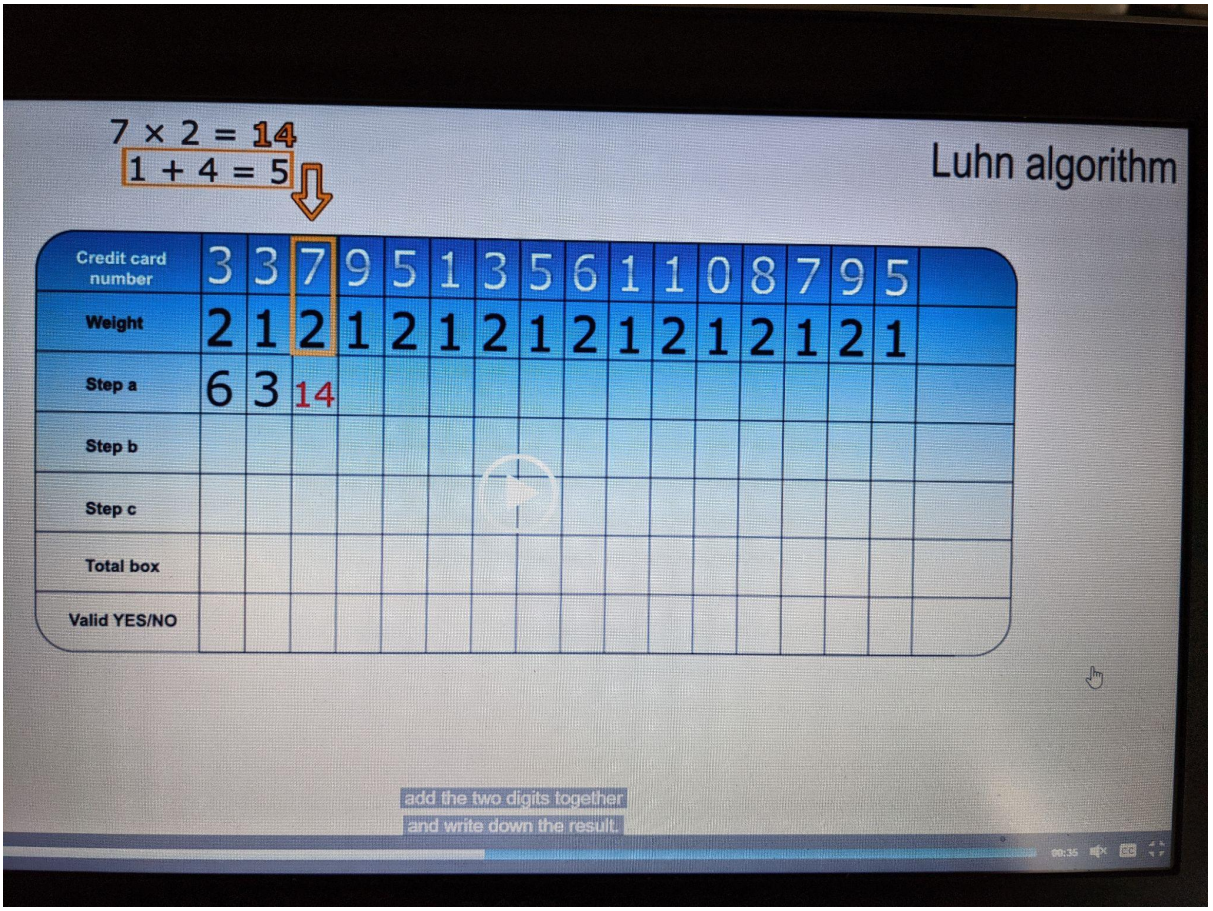

MR T MINAMI
3379 5135 6110 8795
Credit card machine readers check
the validity of the credit card number


Valid number
using a special set of steps
called the Luhn algorithm.


MYP Banking
3379 5135 6110 8795
MR T MINAMI
VALID FROM 05/2017
A Luhn algorithm validity check is made
following these steps:


3 × 2 = 6
Luhn algorithm
Credit card number
Weight
Step a
Step b
Step c
Total box
Valid YES/NO
A. Double every second digit,
starting with the first digit from the left.


7 × 2 = 14
1 + 4 = 5
Luhn algorithm
Credit card number 3 3 7 9 5 1 3 5 6 1 1 0 8 7 9 5
Weight 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
Step a 6 3 14
Step b
Step c
Total box
Valid YES/NO
B. If the double digit is
a two-digit number,

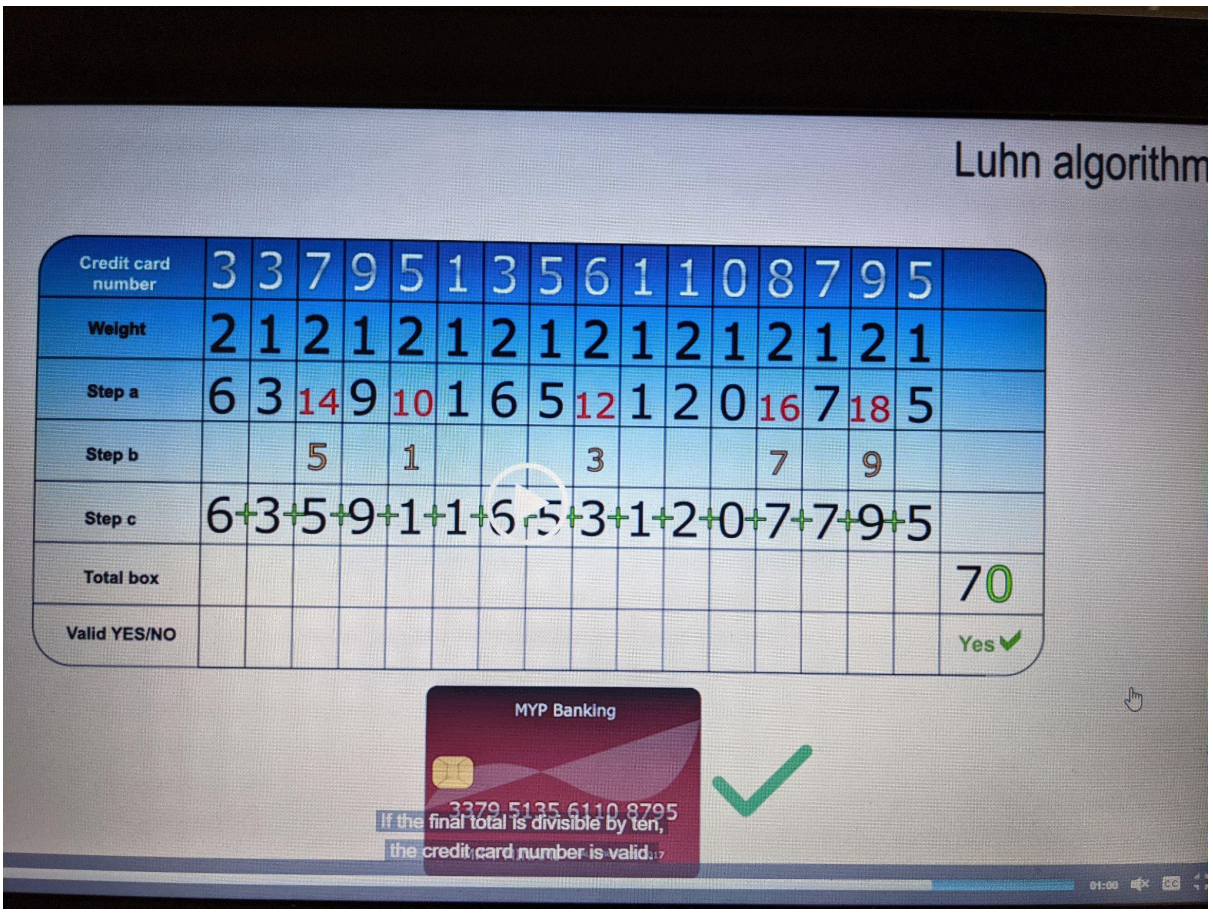

7 × 2 = 14
1 + 4 = 5
Luhn algorithm
Credit card number 3 3 7 9 5 1 3 5 6 1 1 0 8 7 9 5
Weight 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
Step a 6 3 14
Step b
Step c
Total box
Valid YES/NO
add the two digits together
and write down the result.

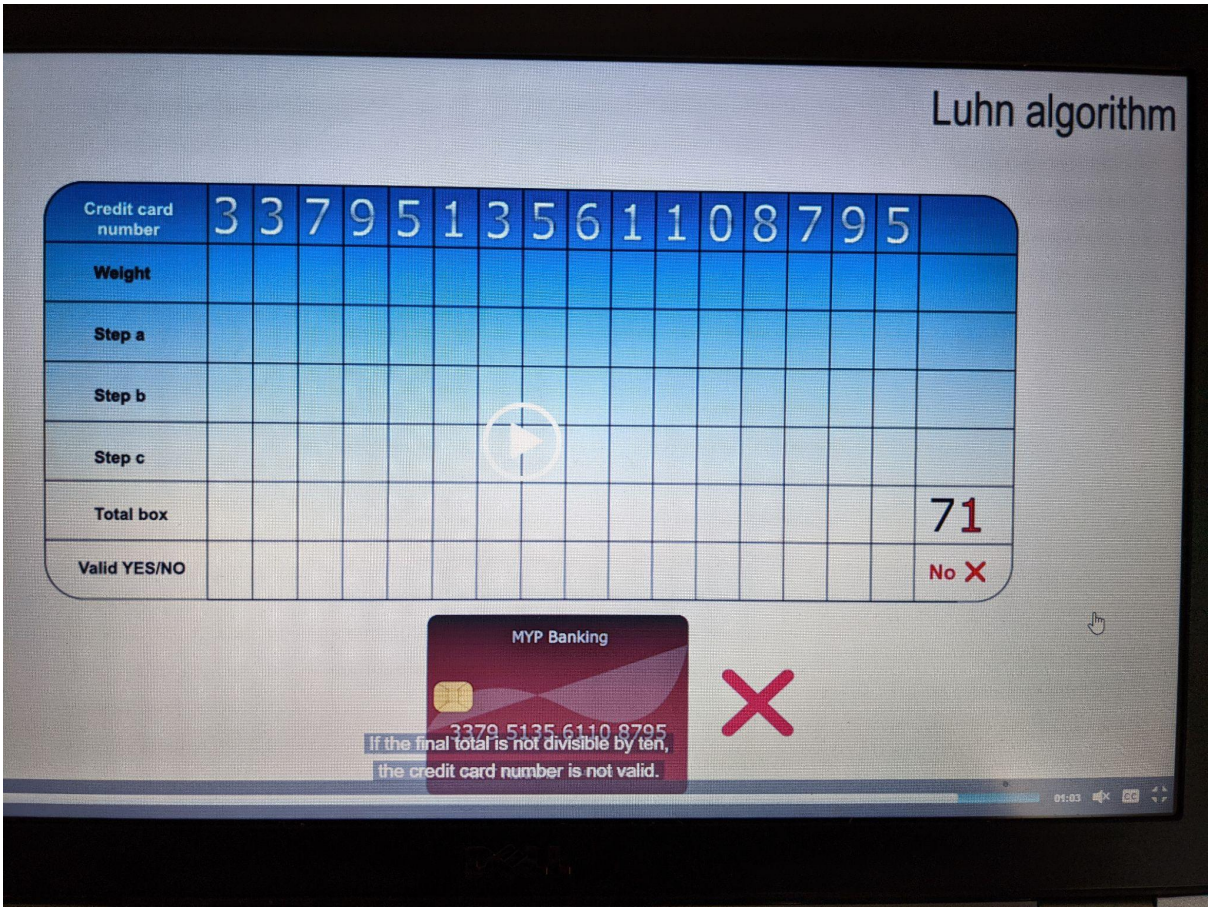
Luhn algorithm

Credit card number	3	3	7	9	5	1	3	5	6	1	1	0	8	7	9	5
Weight	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
Step a	6	3	14	9	10	1	6	5	12	1	2	0	16	7	18	5
Step b			5		1				3				7		9	
Step c	6	3	5	9	1	1	6	5	3	1	2	0	7	7	9	5
Total box																
Valid YES/NO																

C. Add all digits together
to find the final total.

00:38


Luhn algorithm
Credit card number 3 3 7 9 5 1 3 5 6 1 1 0 8 7 9 5
Weight 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1
Step a 6 3 14 9 10 1 6 5 12 1 2 0 16 7 18 5
Step b 5 1 3 7 9
Step c 6+3+5+9+1+1+6+5+3+1+2+0+7+7+9+5
Total box 70
Valid YES/NO Yes
MYP Banking
3379 5135 6110 8795
If the final total is divisible by ten,
the credit card number is valid.


Luhn algorithm
Credit card number 3 3 7 9 5 1 3 5 6 1 1 0 8 7 9 5
Weight
Step a
Step b
Step c
Total box 71
Valid YES/NO No
MYP Banking
3379 5135 6110 8795
If the final total is not divisible by ten,
the credit card number is not valid.

Credit card machine readers check the validity of credit card numbers using a special set of steps called the Luhn algorithm. A Luhn algorithm validity check is made following these steps:

A. Double every second digit, starting with the first digit from the left.
B. If the double digit is a two-digit number, add the two digits together and write down the result.
C. Add all the digits together to get the final total.

If the final total is divisible by ten, the credit card number is valid.
If the final total is not divisible by ten, the credit card number is not valid.

Question 3a (2 marks)

Determine the missing digits represented by the letters A, B, C, D and E to complete the table below.

To insert your answers in the table, click inside the box and replace the letters with your answers in the "Add Label" box.

Credit card number	1	5	4	3	4	1	3	8	7	2	3	5	8	3	4	6
Weight	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
Step a	2	5	8	3	8	1	6	8	A	2	6	5	16	B	8	6
Step b																
Step c	2	5	8	3	8	1	6	8	C	2	6	5	D	E	8	6

Question 3b (2 marks)

Justify why the credit card number is invalid.

Question 3c (2 marks)

The credit card 4150 0811 1727 967X is a valid credit card number.

After the Luhn algorithm is applied, the final total is 61 + X. **Determine** the value of X.

Question 3d (3 marks)

Label the table below with further instructions F, G and H to clarify the workings of the algorithm. Some instructions have already been given.

Credit card number	3379 5135 6110 8795
Weight row	Identify the digits that are doubled
Product	Double the appropriate digits
Instruction F	
Result	Write down the new digit after instruction F
Instruction G	
Instruction H	

Question 4 (7 marks)

The function f(x) is defined for the domain {0, 2, 4, 6}. The values of f(x) are given in the table below.

x	0	2	4	6
f(x)	−4	0	2	3

Question 4a (1 mark)

Write down the value of f(4).

Question 4b (2 marks)

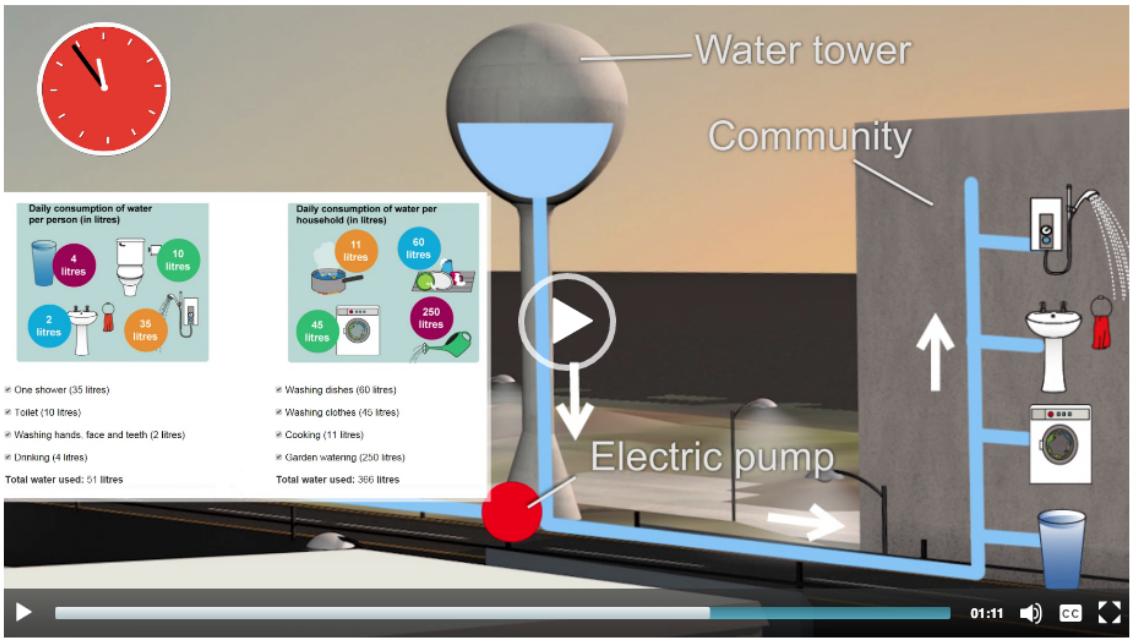
Determine the value of 3f(4) −1.

Question 4c (4 marks)

Determine the value of x if $3 - 2f(x) = 11$.

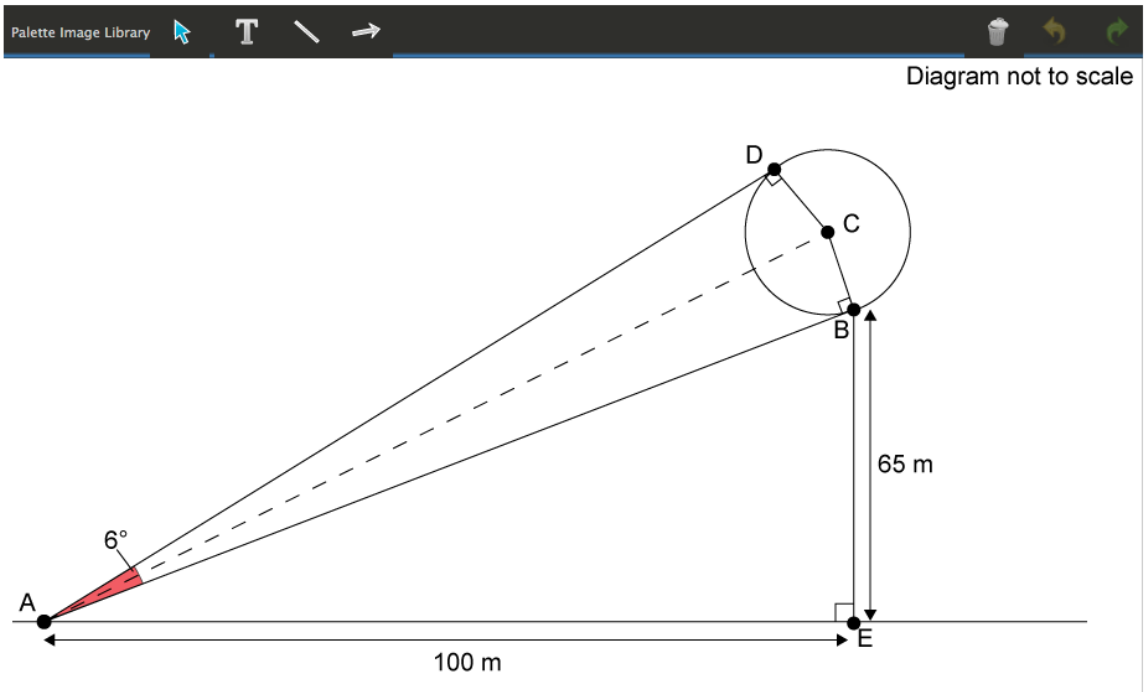
Question 5 (20 marks)

The following video describes how a water tower can be used to maintain residential water pressure for a community.



Question 5a (1 mark)

Diagram 1



The water is stored in a spherical water tank that is modelled by the cross section in Diagram 1, with centre C. The distances are recorded to the nearest metre (m). EB is 65 m and AE is 100 m. Angle BAD is 6° recorded to the nearest degree. AB and AD are tangents to the spherical water tank at points B and D respectively. Angle AEB, ABC and ADC are 90°. ABCD is a kite.

Write down the size of angle BAC.

Question 5b (4 marks)

Show that BC, the length of the radius of the spherical water tank is 6.25 m, to the nearest cm.

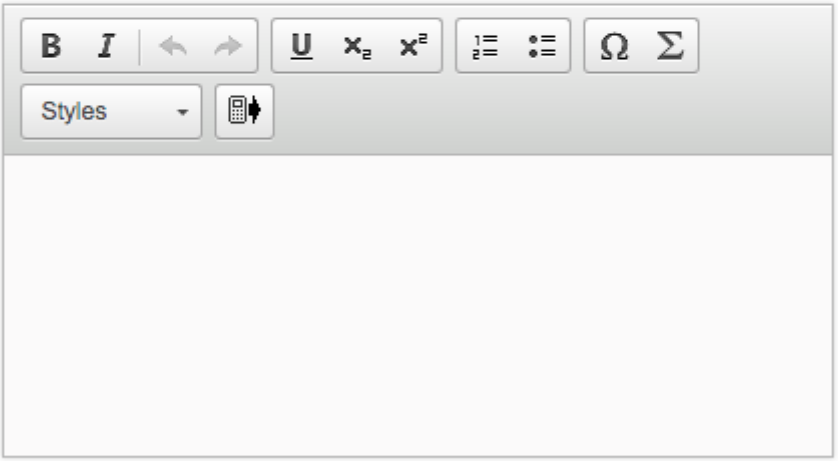
Question 5c (3 marks)

Hence, **calculate** the volume of the spherical water tank to the nearest cubic metre (m^3).

Question 5d (2 marks)

Given that the maximum volume of the water held in the spherical water tank is 950 m^3, **identify** **two** possible reasons for the difference between the volume you calculated and the maximum volume.

Reason 1



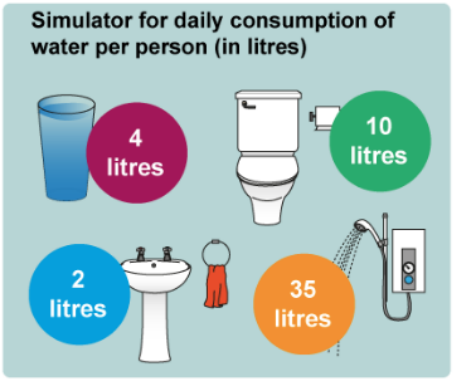
Reason 2

The MYP water tower serves the MYP county area with a population of approximately 300 000 people. On average there are four people per household and the typical water consumption distribution is shown in the simulations below (1000 litres = 1 m^3).

The water tower provides a supply of water for household use during a power outage. There will be a planned power outage for maintenance reasons on Tuesday 30th May 2017 between 9am and 1pm. You are responsible for gathering information on the possible impact to the water supply for the community during the power outage. An interactive simulator is provided for the different household activities.

Use the water consumption simulators below for the different activities.

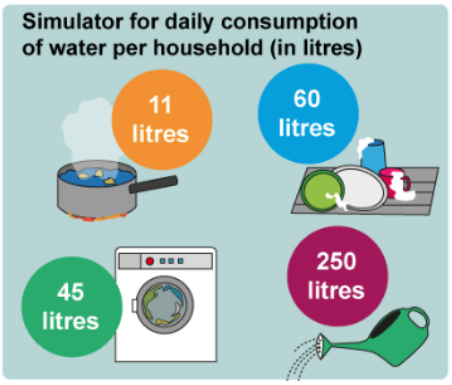
This media is interactive



- [ ] One shower (35 litres)
- [ ] Toilet (10 litres)
- [ ] Washing hands, face and teeth (2 litres)
- [ ] Drinking (4 litres)

Total water used:

This media is interactive



- [ ] Washing dishes (60 litres)
- [ ] Washing clothes (45 litres)
- [ ] Cooking (11 litres)
- [ ] Garden watering (250 litres)

Total water used:

Question 5e (10 marks)

Discuss the implications on water consumption for households in the community during this time.

In your answer you should:

- calculate the consumption in litres per hour (l/h) for the households and individuals in the community
- estimate the amount of time before the water (950 m^3) held in the tank runs out
- advise the community about the activities that should be avoided during the power outage
- justify the degree of accuracy of the time calculation.

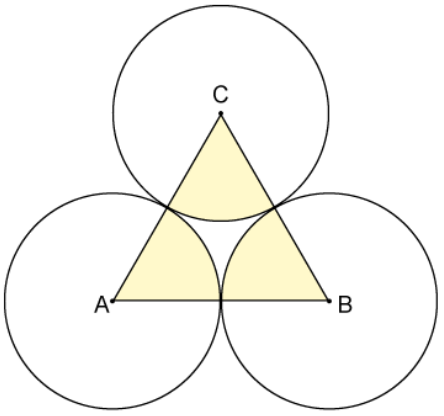
Question 6 (10 marks)

The following video explains the requirements for a reliable mobile phone signal.
Mobile phones are also known as cell phones.

In Diagram 1, identical transmitters are positioned at the vertices of an equilateral triangle. The equilateral triangle has sides of length 20 km and area 173 km^2. The transmitters are placed at A, B and C, and they emit a frequency in a circular radius up to 10 km.

Diagram 1

Diagram not to scale



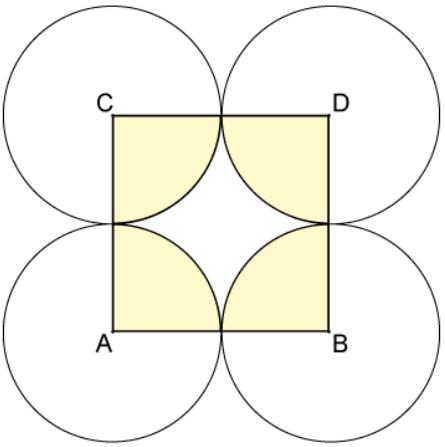
Question 6a (4 marks)

The unshaded area inside the triangle represents the part of the triangle that receives no signal.
Find the area of the triangle that does not receive a signal.

In Diagram 2, identical transmitters are positioned at the vertices of a square. The square has sides of length 20 km and area 400 km^2. The transmitters are placed at A, B, C and D, and they emit a frequency in a circular radius up to 10 km.

Diagram 2

Diagram not to scale



Question 6b (2 marks)

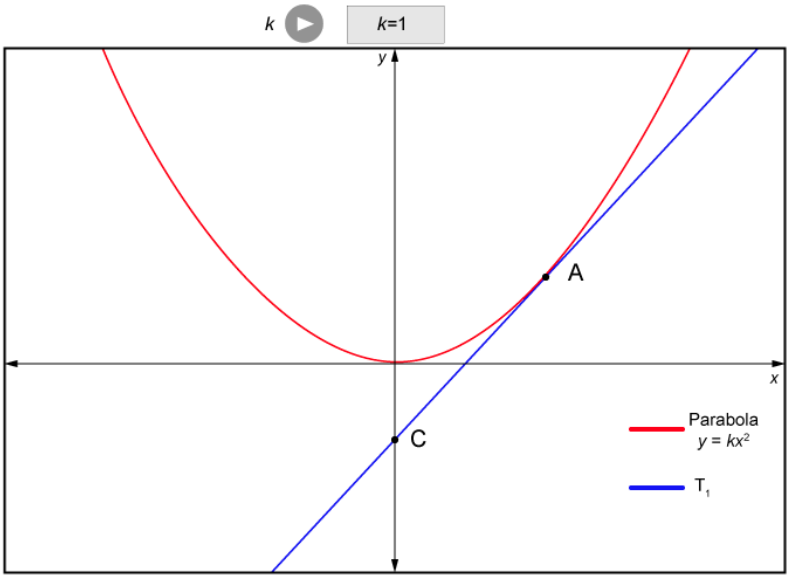
The unshaded area inside the square represents the part of the square that receives no signal.
Find the area of the square that does not receive a signal.

Question 6c (4 marks)

Compare your answers in parts (a) and (b). In your answer, refer to the different ways the transmitters have been positioned and propose the most suitable layout for the most efficient use of space and best signal coverage.

Question 7 (35 marks)

The straight line T_1 touches the parabola $y = kx^2$ once at point A as shown in the simulation below where k is the coefficient of x^2. As the value of k changes, the point A changes but the gradient of T_1 stays as 1. Click on the arrow to see what happens to the coordinate A as the value of k changes.



Question 7a (2 marks)

As the value of k changes, the following coordinates for point A are recorded in the table below.

If you would like to add more values in the table, click inside the relevant box, then write the values in the "Add Label" box.

k	1	2	3	4	5	6			
x coordinate of point A (x_A)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$
y coordinate of point A (y_A)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$

Describe in words a pattern for x_A.

Question 7b (1 mark)

Predict the value of x_A for $k = 6$.

Question 7c (1 mark)

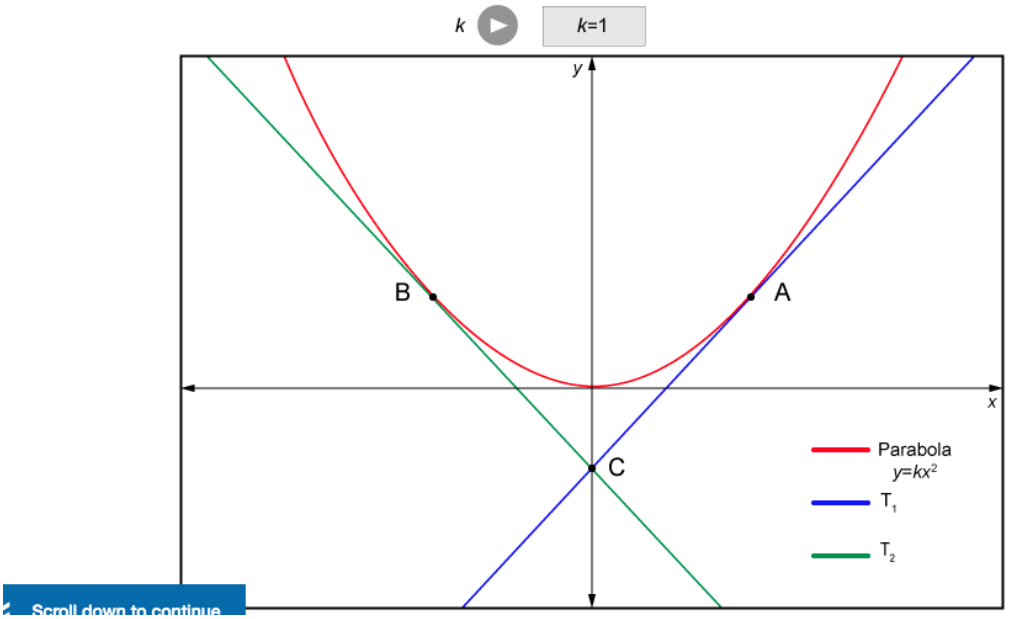
Write down a general rule for x_A in terms of k.

Question 7d (3 marks)

Verify your general rule for x_A.

Using the simulation below for the parabola $y = kx^2$, click on the arrow to see what happens as the value of k, the coefficient of x^2 changes. The lines T_1 and T_2 touch the parabola once as shown in the diagram at points A and B respectively. T_1 and T_2 intersect at the point C on the y-axis.

The coordinates of A, B and C change with k. The gradient of T_1 is fixed at 1 and the gradient of T_2 is fixed at -1. The lines T_1 and T_2 are perpendicular. The line T_1 has equation $y = x + c$.



Scroll down to continue

Question 7e (2 marks)

As the value of k changes, the following coordinates for A, B, and C have been recorded in the table below.

k	1	2	3	4	5	
x coordinate of point A (x_A)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{\square}$
y coordinate of point A (y_A)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{20}$	$\frac{1}{\square}$
x coordinate of point B (x_B)	$\frac{-1}{2}$	$\frac{-1}{4}$	$\frac{-1}{\square}$	$\frac{-1}{\square}$	$\frac{-1}{\square}$	$\frac{-1}{\square}$
y coordinate of point B (y_B)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$	$\frac{1}{\square}$
y coordinate of point C (y_C)	$\frac{-1}{4}$	$\frac{-1}{8}$	$\frac{-1}{\square}$	$\frac{-1}{\square}$	$\frac{-1}{\square}$	$\frac{-1}{\square}$

Determine the missing values and complete the table for $k = 3$ and $k = 4$.

To add the missing values in the table, for $k = 3$ and $k = 4$, click inside the relevant box, then write the values in the "Add Label" box.

Question 7f (4 marks)

Describe two patterns for y_A, and **two** patterns for x_B.

y_A

Question 7f (4 marks)

Describe **two** patterns for y_A, and **two** patterns for x_B.

y_A

B I U x_2 x^2 Ω Σ Styles

x_B

Question 7g (22 marks)

Investigate the relationship between y_C and k.

In your answer you should:

- make predictions for more values of k
- describe the pattern
- find a general rule for y_C in terms of k
- test your general rule
- prove or verify and justify your general rule
- ensure you communicate the above appropriately.